# Analysis of a Continuous Rotary-Drum Filtration System

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*The equation traditionally used to analyze a rotary-drum vacuum filtration system is strictly valid only for flat filters immersed during the filtration cycle. In this article we derive the equation appropriate for a rotary-drum filter and find that the results of calculated areas differ by a factor 5 from the fully immersed filter result.*  AIChE J, 56: 1737–1738, 2010
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## Introduction

The derivation of the equations governing filtration rates of vacuum filter cakes is available in many texts, a recent one of which was written by C. J. Geankoplis[1] and will be followed here (Chapter 14).

If a flat vacuum filter of area A is in contact with a slurry, then the volumetric rate, $dV/dt$, that filtrate passes through the filter cake obeys the equation

$$\frac{dV}{Adt} = \frac{-\Delta p}{\mu(\alpha c_s V/A + R_m)} \tag{1}$$

where $\Delta p$ is the pressure drop across the cake; $R_m$, the resistance of the filter medium to filtrate flow; $\alpha$, the specific cake resistance; $c_s$, the mass of solids per volume of filtrate in the slurry; and $\mu$, the viscosity of the filtrate.

Assuming the filtration is begun at $t = 0$ and is run until $t = t_f$, then the integration of Eq. 1 from 0 to $t_f$ and solution for $V/A$ yields the equation

$$\frac{V}{A} = \frac{-R_m + \sqrt{R_m^2 + 2\alpha c_s(-\Delta p)t_f/\mu}}{\alpha c_s} \tag{2}$$

Several texts, including Geankoplis,[1] Rushton, Ward and Holdich,[2] and Purchas[3] apply Eq. 2 in analyzing the production rate of a rotary-drum vacuum filtration system. We have been unable to find a source doing otherwise. However, Eq. 2 does not apply directly to the rotary-drum vacuum filtration system. Equation 2 refers to a filter in which the entire area $A$ is in the slurry the entire time $t_f$. The purpose of this short article is to present the correct equations for the rotary-drum vacuum filtration system.

## Analysis of the Rotary-Drum Filter

For our purpose, the diagram of a vacuum-drum filter is shown in Figure 1. The vacuum is pulled in the drum. There is a wash cycle not indicated in Figure 1. The filter cake removal shown in the diagram is done with a doctor blade. There are other ways to discharge the filter cake. For better diagrams and pictures of rotary-drum filters, see Refs. 1–3 and literature quoted in these references.

The drum rotates with a rotational velocity of $\omega = 1/t_c$, where $t_c$ is the time for the drum to make one full cycle. The part of the drum in contact with the slurry passes from $\theta = 0$ to $\theta = \theta_f$ in the time $t_f$, where $t_f = ft_c$. $f$ is the fraction of the rotational time $t_c$ that a point on the drum entering the slurry at $\theta = 0$ takes to leave the slurry at $\theta_f$.

Shown in the figure is an element of area

$$\delta A_i = WR\delta\theta_i, \tag{3}$$

where $W$ is the width of the drum and $R$ is its radius. The time it takes the element of area to travel from $\theta = 0$ to $\theta_i$ is given by

$$t_i = \frac{\theta_i}{2\pi\omega}, \tag{4}$$

where $t_i$ is the time it takes a point on the drum to rotate by an angle $\theta_i$.

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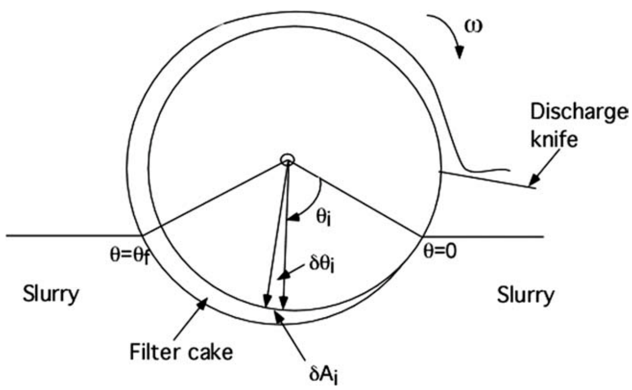


**Figure 1. Schematic of a continuous rotary-drum filter. (See Fig. 14.2–5, p 909, in Ref. 1).**

Suppose the element of area $\delta A_i$ began at $\theta = 0$ at $t = 0$. When it arrives at $\theta_i$ it will have been in the slurry for the time $t_i$, where $\theta_i = 2\pi\omega t_i$. According to Eq. 2, the volume $\delta V_i$ of filtrate that will have passed through $\delta A_i$ is given by

$$\frac{\delta V_i}{\delta A_i} = \frac{\{-R_m + [R_m^2 + 2c_s\alpha(-\Delta p)t_i/\mu]^{1/2}\}}{\alpha c_s}. \quad (5)$$

as $\delta\theta_i = 2\pi\omega\delta t_i$, we can rewrite Eq. 5 as

$$\delta V_i = \left\{\frac{-R_m + [R_m^2 + 2c_s\alpha(-\Delta p)t_i/\mu]^{1/2}}{\alpha c_s}\right\} 2\pi WR\omega\delta t_i. \quad (6)$$

Noting that $A = 2\pi WR$, where $A$ is the area of the drum, and passing to infinitesimal $dV$ and $dt$ and integrating Eq. 6 from $t = 0$ to $t = t_f$, we obtain for the volume $V$ of filtrate passing through the cake in one cycle the formula

$$V = \left\{-\frac{R_m}{\alpha c_s} + \frac{2[R_m^2 + 2c_s\alpha(-\Delta p)t_f/\mu]^{3/2} - 2R_m^3}{3\alpha c_s[2c_s\alpha(-\Delta p)/\mu]t_f}\right\} Af. \quad (7)$$

For negligible values of $R_m$ or sufficiently large values of $t_f$, Eq. 7 for rotary drums can be approximated as

$$V = \frac{2}{3\alpha c_s}(2\alpha c_s(-\Delta p)/\mu)^{1/2} f^{3/2} t_c^{1/2} A. \quad (8)$$

The equivalent approximation to Eq. 2 for a fully immersed filter is

$$V = \frac{1}{\alpha c_s}(2\alpha c_s(-\Delta p)/\mu)^{1/2} f^{1/2} t_c^{1/2} A. \quad (9)$$

Thus, for negligible $R_m$, we have the simple scaling law

$$\frac{V_2}{A_2} = \frac{2f}{3}\frac{V_1}{A_1}, \quad (10)$$

where $V_2$ and $A_2$ are the filtrate volume and area of the rotary drum filter, and $V_1$ and $A_1$ are the filtrate volume and area of the test-leaf filter for a run of time $t_f$. Typically, $f$ is chosen to be of order 1/3, and so for equal volumes of filtrate, $V_2 = V_1$, the area of the rotary drum filter would have to be 4.5 times that of a test-leaf (or any flat, immersed filter) filter.

Even for non-negligible values of $R_m$, one can expect the drum area to be much larger than a leaf-type filter. For example, consider some typical data from a test-leaf filter experiment on a 4 wt % slurry of celite 500 filter aid:

$$R_m = 2.1 \times 10^{11}\text{m}^{-1}, \alpha = 1.8 \times 10^{11}\text{m}/kg,$$
$$\rho = 1000\ kg\text{slurry}/m^3\text{slurry}, c_x = 0.04\ kg\text{solids}/1kg\text{slurry}$$
$$c_s = 47.9\ kg\text{solids}/m^3\text{filtrate}, -\Delta p = 60948\ Pa,$$
$$\mu = 0.0013 Pa.s. \quad (11)$$

Suppose, we want to filter $4.2 \times 10^{-3}$ m$^3$ of aqueous slurry per second. Then, the filtrate volume $V$ per unit cycle of the rotating drum filter is

$$\frac{V}{t_c} = 3.5 \times 10^{-4}\frac{\text{m}^3}{\text{s}} \quad (12)$$

We choose a cycle time $t_c = 785$s and slurry contact time $t_f = 235.5 t_c$ or $f = 0.3$. Calculating $A_2$ from Eq. 7, we obtain

$$A_2 = 50.9\text{m}^2 \quad (13)$$

From Eq. 2, we obtain

$$A_1 = 8.64\text{m}^2, \quad (14)$$

or

$$\frac{A_2}{A_1} = 5.89. \quad (15)$$

From the approximation at Eq. 10, we find

$$\frac{A_2}{A_1} = 5, \quad (16)$$

surprisingly close to the more accurate calculation that includes $R_m$, Eq. 13.

In any case, Eq. 2 is quite inaccurate for estimating the area needed for a rotary drum filtration system to provide a given filtration rate.

From the calculation here, we recommend as the simplest method of choosing a rotary-drum vacuum filter system to deliver a given filtrate production $V_2$ that a bench-top test-leaf filter of area $A_1$ be used to determine $\alpha$, $R_m$ and $V_1$ for a given slurry and filter cloth and a given pressure drop $\Delta p$ and filter time $t_f$. Then, Eq. 10 can be used to give a rough estimate of the area $A_2$ of the rotary-drum filter, or Eq. 7 can be used for a more precise estimate.

## Literature Cited


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